# Tilt Series Reconstruction and Parallax Problem

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The recent development of a *hardware* aberration corrector for transmission electron microscope has significantly improved the attainable resolution [1]. It has demonstrated that imaging with a negative Cs gives higher contrast than a positive Cs case, and improves sensitivity to detect a light element [2]. On the other hand, aberration can be *a posteriori* corrected when we reconstruct complex wave function from a series of through-focus images taken with a non-Cs corrected microscope [3,4]. Importantly, a focal series reconstruction (FSR) gives an aberration-corrected *complex* wave function at the specimen exit surface, but a hardware aberration corrector gives only *intensity* distribution of the wave function. However, aberration corrected image obtained by using hardware or software is ultimately limited by the partial coherence of electrons (information limit). A promising method to improve resolution beyond the information limit is a tilt series reconstruction (TSR), where several images are recorded with different beam tilt [5]. Since this is 'aperture synthesis', we can improve resolution substantially. The sophisticated TSR procedure including a short focal series has been already developed [6], and is commercially available [7]. However, it has been argued that a serious limitation is imposed on specimen thickness by a parallax problem due to the use of tilted illumination. We will study the previous discussion for the geometric parallax, and derive a new estimate of an allowable specimen thickness in the case of the tilted illumination.

Under tilted illumination displacement between the images (geometrical parallax) of the points on the top and bottom specimen surfaces will be $t \cdot \tau$, where $t$ is the specimen thickness and $\tau$ a tilt angle. In order to resolve a periodicity $d$ the geometrical parallax may be limited by $d/2$. Thus, a maximum thickness imposed by the geometrical parallax will be $d/2\tau$. The same conclusion has been derived using the phase shift between the points on the top and bottom specimen surfaces due to a beam tilt [8]. We may note that this formula is independent of an accelerating voltage (or a wavelength), and gives infinite thickness for a normal incident. We will verify that this formula corresponds to the high-accelerating-voltage limit, where the wavelength decreases to zero and the Ewald sphere becomes flat.

We discuss here the parallax problem by taking into account physical scattering with a thin sample of thickness $t$. Figure 1 illustrates the Ewald construction for a tilted illumination, where we assume the diffraction plane (the zero order Laue zone) is parallel to the specimen slab. Here, $\tau$ is the tilt angle, $\lambda^*(1/\lambda)$ the radius of the Ewald sphere, $g_{\max}$ the resolution limit. In kinematical approximation the scattering distribution in reciprocal space will elongate approximately by $1/t$ perpendicular to the specimen slab. The Ewald sphere and the diffraction plane intersect at $g$, and $\varsigma$ and $\varsigma_\tau$ correspond to excitation errors (distances between the diffraction spot center and the Ewald sphere) for scattering vectors $g_{\max}$ and $g/2$, respectively. We may note two relationships $\tau \approx \lambda\left(g/2\right)$ for a small tilt, and $2\varsigma \le 1/t$ at $g_{\max}$. The following two equations will be observed, when we apply Pythagorean theorem to the triangles $\Delta$EOA and $\Delta$ECB:

$$\left(\lambda^* - \varsigma_\tau\right)^2 + \left(g/2\right)^2 = \left(\lambda^*\right)^2 \quad \text{and} \quad \left(\lambda^* - \varsigma_\tau - \varsigma\right)^2 + \tilde{g}_{\max}^2 = \left(\lambda^*\right)^2$$

where $\tilde{g}_{\max} = g_{\max} - (g/2)$. Then, they respectively reduce to the following two approximations:

$$(g/2)^2 \cong 2\varsigma_\tau \lambda^* \quad \text{and} \quad (\tilde{g}_{\max})^2 \cong 2(\varsigma + \varsigma_\tau)\lambda^*$$

Using these approximations and the above-mentioned two relationships we finally get the expression for the maximum thickness as a function of the beam tilt τ and the resolution $g_{\max} : t \le 1 / \left(\lambda g_{\max}^2 - 2 g_{\max} \tau\right)$. This is a general formula and the geometrical parallax is deduced at the limit $\lambda \to 0$. Furthermore, this gives a correct thickness limit for the normal incidence: $t \le 1/\lambda g_{\max}^2$. For a given $g_{\max}$ the two excitation errors become equal ($\varsigma_\tau = \varsigma$) at the optimum tilt angle, and the maximum thickness is given as $t \le \left(3 + 2\sqrt{2}\right) / \lambda g_{\max}^2$. Thus, for the same resolution $g_{\max}$ the TSR can be applied to a 5.8-times thicker sample than the FSR. Even when we improve resolution using the TSR twice of the resolution for the normal illumination, we can still use a 1.5-times thicker sample than the FSR. These conclusions may be surprised in terms of the geometrical parallax. However, the TSR collects information on a diffraction plane over a wider area by an aperture synthesis. Then, using the Fourier projection theorem the TSR gives a better projection of a sample. Thus, the parallax due to sample thickness may not be a limiting factor for the TSR.

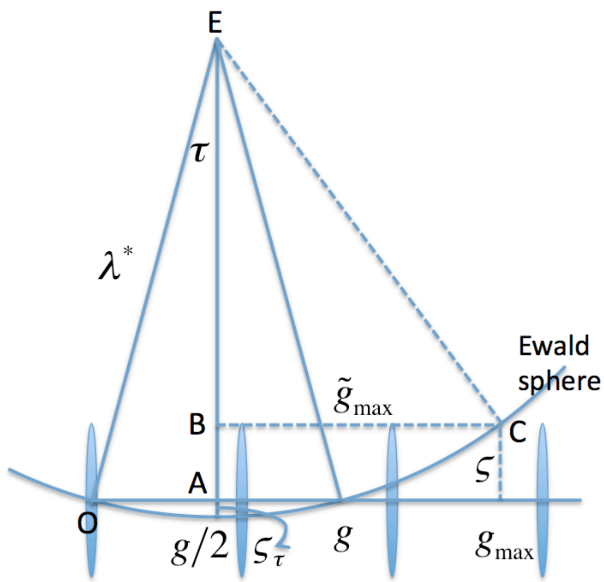


**Figure 1.** Ewald construction for a tilted illumination. Here, τ is the tilt angle, $\lambda^*$ the radius of the Ewald sphere, $g_{\max}$ the resolution limit and $\varsigma$ and $\varsigma_\tau$ excitation errors. The scattering distribution in reciprocal space elongates approximately by 1/thickness perpendicular to the specimen slab.

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