# Optimal Noise Filters in High-Resolution Electron Microscopy

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Most of the specimens for high-resolution electron microscopy have amorphous surface layers due to contamination or damaged surface layers due to specimen preparation. Moreover, many specimens are radiation sensitive, and a part of the specimen becomes amorphous during the observation. These amorphous materials hinder clear observation of crystal structure. Kilaas discussed the optimal Wiener filter and near-optimal background subtraction filter to extract crystal information [1]. However, these filters do not work for non-ideal crystals, such as cylindrical crystals and nano-crystals. We propose here two techniques that make these filters applicable even for such materials.

We may write an observed signal $F_o$ in Fourier transform as a sum of a true signal $F_c$ due to a crystal part and a background $F_b$ due to a non-crystal part: $F_o = F_c + F_b$. The Wiener filter seeks a solution that minimizes the summed square difference between the true signal $F_c$ and its estimate $\hat{F}_c$ resulting

$$\hat{F}_c = \frac{|F_c|^2}{|F_c|^2 + |F_b|^2} F_o \approx \frac{|F_o|^2 - |\hat{F}_b|^2}{|F_o|^2} F_o = \frac{|F_o|^2 - |\hat{F}_b|^2}{|F_o|} e^{i\phi_o},$$

while the background subtraction filter (we call here the Difference filter) is given

$$\hat{F}_c = \left(|F_o| - |\hat{F}_b|\right) e^{i\phi_o},$$

where $\phi_o$ is the phase of the observed signal $F_o$ and $\hat{F}_b$ the estimate of the background. If $|F_o| - |\hat{F}_b| \le 0$, $\hat{F}_c$ is set to zero.

Normally, the background is estimated as a radial average of the Fourier transform of the whole image assuming that the contribution from amorphous (non-periodic) materials varies slowly. Here, we use an image taken from crysotile, a clay mineral (Fig. 1a). Fig.1b is a trend-subtracted image, where you can see more easily a specimen structure, while Fig. 1c a Fourier transform of the original image. A Wiener filtered image using a radial average background is reproduced in Fig. 2a, which is seemingly good. However, its difference to the original image clearly demonstrates an insufficient extraction of the structure (Fig. 2b). The Fourier transform of the difference shows extraordinary rings due to the radial average background (Fig. 2c). The reason why the radial Wiener filter fails to extract the structure is that the signal itself appears at about equidistant from the origin as shown in Fig. 1c.

Thus, we developed a novel approach to estimate a smoothed two-dimensional background based on P-spline fitting [2], and reported elsewhere [3]. Fig. 3a shows a Wiener filtered image using a two-dimensional background. The difference to the original image clearly shows substantial improvement to extract the structure, although a faint structure residue is discernible (Fig. 3b). You may note that the difference is anisotropic as shown in Fig. 3c. This indicates that we cannot use a background obtained from the whole image.

Fig. 4a shows a Wiener filtered image using a set of two-dimensional backgrounds estimated over each 64x64 pixels. The difference to the original image (Fig. 4b) and its Fourier transform (Fig. 4c) do not show any structural features. We call this filter Local 2D Wiener filter. This filter is also useful to extract nano-crystals embedded in an amorphous substrate, like $TiO_2$ (anatase) in amorphous silica [4].

Refarences

[1] R. Kilaas, *J. Microscopy* 190 (1997) 45-51.
[2] P.H.C. Eilers et al, *Computational Statistics and Data Analysis* 50 (2006) 61-76.
[3] P.H.C. Eilers and K. Ishizuka, IMC16, Sapporo, (2006) 964.
[4] T. Kogure, P.H.C. Eilers and K. Ishizuka, Japanese EM Meeting, Niigata (2007).

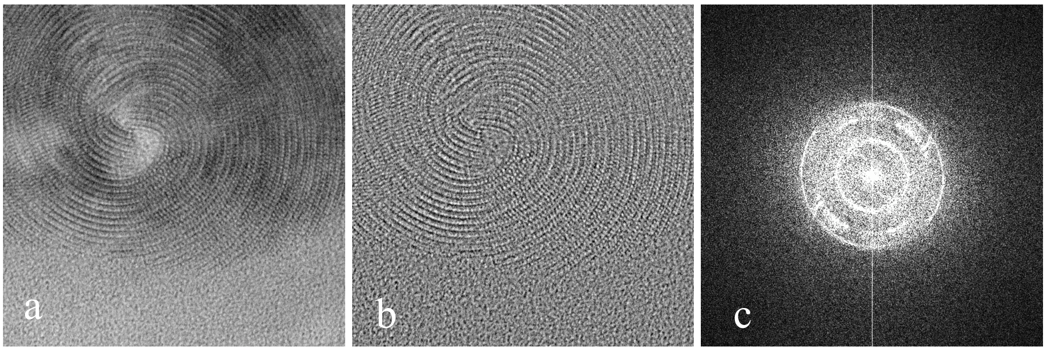

Fig. 1 (a) Original image (crysotile), (b) trend-subtracted image, (c) FT of the original image.

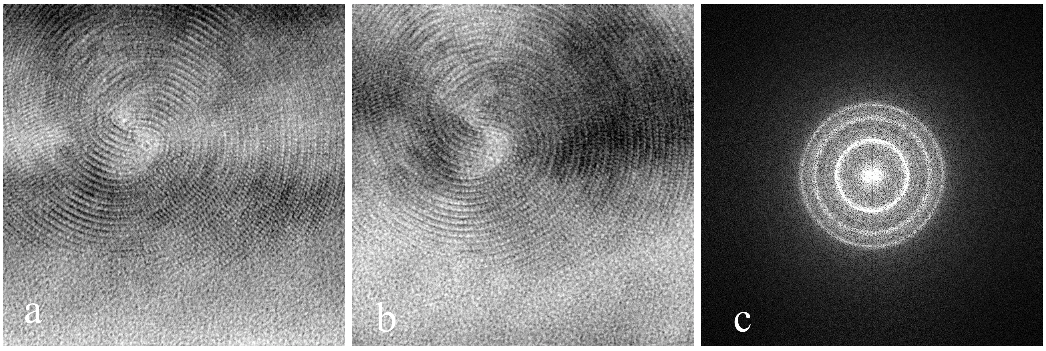

Fig. 2 (a) Wiener filtered image with a radial background, (b) difference wrt the original image, (c) FT of the difference.

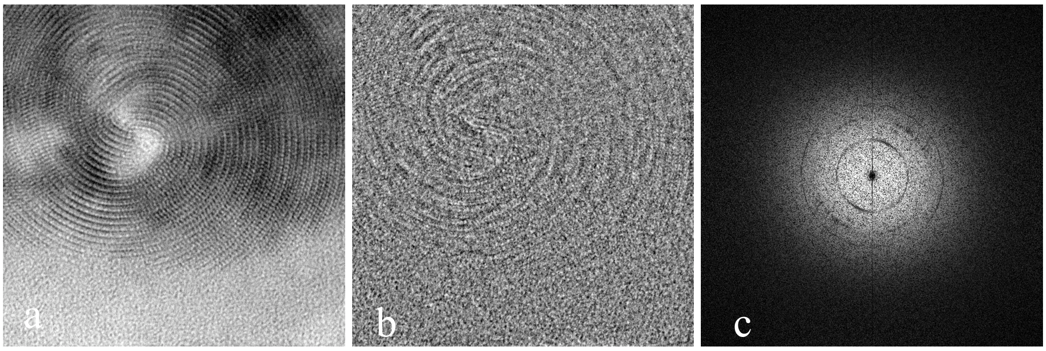

Fig. 3 (a) Wiener filtered image with a 2D background, (b) difference wrt the original image, (c) FT of the difference.

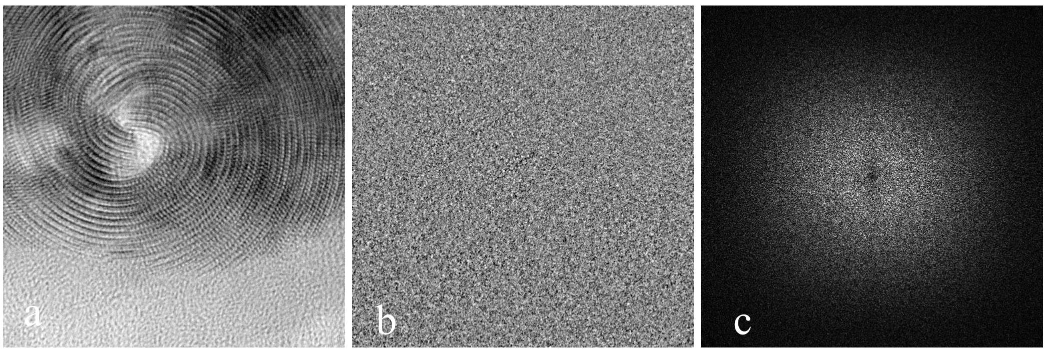

Fig. 4 (a) Wiener filtered image with a local 2D background, (b) difference wrt the original image, (c) FT of the difference.